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Real-time state observers based on multibody models and the extended Kalman filter †

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Abstract

This work is a preliminary study on the use of the extended Kalman filter (EKF) for the state estimation of multibody systems. The observers based on the EKF are described by first-order differential equations, with independent, non-constrained coordinates. Therefore, it should be investigated how to formulate the equations of motion of the multibody systems so that efficient, robust and accurate observers can be derived, which can serve to develop advanced real-time applications. In the paper, two options are considered: a state-space reduction method and the penalty method. Both methods are tested on a four-bar mechanism with a linear spring-damper. The results enable us to analyze the pros and cons of each method and provide clues for future research.

Keywords: Multibody dynamics; Real-time applications; State estimation; Vehicle controllers

1. Introduction

The extended Kalman Filter (EKF) has been widely used in combination with nonlinear dynamic models of systems as state observer in several fields.

In current practice, the EKF is combined with simplified dynamic models of the systems and elementary numerical integration schemes in order to streamline convergence and to achieve real-time performance of the computation process.

However, current state-of-the-art knowledge in multibody dynamics opens the possibility of considering complex multibody models in real-time state observer applications, as long as specialized schemes are employed [1]. The advantage is that more information can be extracted from the model.

The EKF is typically formulated for first order nonlinear systems and non-constrained coordinates, in state-space form (ordinary differential equations, ODE). However, the equations of motion of a multibody system are second order equations of constrained coordinates (differential algebraic equations, DAE).

In the applications reported in the literature, the combination of the EKF with constrained DAE plants is usually addressed from the EKF point of view. That means adapting the KF rationale to the specific DAE problem. For example, in [2] it is shown that the descriptor dynamics give rise to singular measurement noise covariance, and an extended maximum-likelihood method is applied. This same idea is followed in [3], where the constraint (unit quaternion norm) is treated as a pseudo-measurement. In [4], the error from constraint linearization is treated in a separate step, after the EKF, increasing the computational complexity.

In this work, the solution to the combination of EKF and DAE is approached from the DAE point of view. As any observer runs in real-time a copy of the plant, the same techniques that are useful for model-

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ing and fast simulation of complex multibody systems, will also be useful for implementing observers for such systems. In particular, this work reports the EKF formal derivation in the case of a state-space reduction method and in the case of the penalty method.

Although the final objective of the project is to address complex multibody systems in industrial applications, in this first preliminary work a simple example is considered for clarity. This test example is a four-bar mechanism with a spring-damper element, so that conclusions based on this simple system can later serve to address larger and more complex systems.

Two computational versions of the mechanism are created: the first one represents the real "prototype", while the second one plays the role of the "model". To test the observer, the model is not an exact replica of the prototype, but differs in some physical parameters; also, the readings coming from sensors and actuators may be altered when passed to the model. The objective is that the model follows the motion of the prototype with the help of an EKF. Preliminary numerical results and practical discussions are presented at the end of the paper.

2. EKF observer

Consider a nonlinear system (plant) given by:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \boldsymbol{\delta} \\ \mathbf{y} &= \mathbf{h}(\mathbf{x}) + \boldsymbol{\varepsilon} \end{aligned} \tag{1}$$

where $\mathbf{x}$ is the (unknown) state vector, and $\mathbf{y}$ is the known measurements vector. The functions $\mathbf{f}$ and $\mathbf{h}$ are also known, and the equations are affected by state and measurement noises $\boldsymbol{\delta}$, $\boldsymbol{\varepsilon}$, with zero mean and given covariances $\boldsymbol{\Theta}$, $\boldsymbol{\Xi}$, respectively. Then, the EKF is given by [5]:

$$\begin{aligned} \dot{\hat{\mathbf{x}}} &= \mathbf{f}(\hat{\mathbf{x}}, \mathbf{u}) + \mathfrak{K}\left(\mathbf{y} - \mathbf{h}(\hat{\mathbf{x}})\right) \\ \mathfrak{K} &= \mathbf{P}\mathfrak{C}^{\mathrm{T}}\boldsymbol{\Xi}^{-1} \\ \dot{\mathbf{P}} &= \mathfrak{A}\mathbf{P} + \mathbf{P}\mathfrak{A}^{\mathrm{T}} - \mathbf{P}\mathfrak{C}^{\mathrm{T}}\boldsymbol{\Xi}^{-1}\mathfrak{C}\mathbf{P} + \boldsymbol{\Theta} \end{aligned} \tag{2}$$

being the matrices $\mathfrak{A}$, $\mathfrak{C}$, computed as the Jacobians of $\mathbf{f}$ and $\mathbf{h}$ with respect to the states, and evaluated at the estimated trajectory. The EKF locally minimizes the covariance $\mathbf{P}$ of the state-estimation error.

3. Multibody dynamics

In its most basic form, the dynamics of a multibody system is described by the constrained Lagrangian equations:

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{q}} + \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\lambda} &= \mathbf{Q} \\ \boldsymbol{\Phi} &= \mathbf{0} \end{aligned} \tag{3}$$

where $\mathbf{M}$ is the positive semidefinite mass matrix, $\ddot{\mathbf{q}}$ the accelerations vector, $\boldsymbol{\Phi}$ the constraints vector, $\boldsymbol{\Phi}_{\mathbf{q}}$ the Jacobian matrix of the constraints, $\boldsymbol{\lambda}$ the Lagrange multipliers vector, and $\mathbf{Q}$ the applied forces vector.

To adopt the form of the Eqs. (1) required for application of the EKF, the second order system of Eqs. (3) can be written as a first order one, just by doing $\mathbf{x}^{\mathrm{T}} = \{\mathbf{q}^{\mathrm{T}} \quad \mathbf{v}^{\mathrm{T}}\}$ with $\mathbf{v} = \dot{\mathbf{q}}$,

$$\begin{aligned} \dot{\mathbf{q}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= \mathbf{M}^{-1}\left(\mathbf{Q} - \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\lambda}\right) \end{aligned} \tag{4}$$

or, more compactly,

$$\begin{Bmatrix} \dot{\mathbf{q}} \\ \dot{\mathbf{v}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{v} \\ \mathbf{M}^{-1}\left(\mathbf{Q} - \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\lambda}\right) \end{Bmatrix} \quad \Rightarrow \quad \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \tag{5}$$

with the positions and velocities subject to the constraints at position and velocity level,

$$\boldsymbol{\Phi} = \mathbf{0} \quad ; \quad \boldsymbol{\Phi}_{\mathbf{q}}\mathbf{v} = \mathbf{0} \tag{6}$$

If n_d is the number of dependent variables and n_i is the number of degrees of freedom (independent variables) of the multibody system, the size of the problem is $2n_d$ (since the states are positions plus velocities).

However, to match (5) to (1), there are several problems. On the one hand, the Lagrange multipliers are unknowns, and the mass matrix is not always invertible. On the other hand, the formalism (1) does not consider constraints among the states.

In our approach, two formulations that convert the DAE (3) into an ODE have been used: a state-space reduction method known as matrix-**R** method [6] and the penalty method. The derivation of EKF observers for multibody systems based on the two mentioned methods is reported in the two following sections.

4. Matrix-R formulation

The main idea in this method [6] is to obtain an

ODE with dimension n_i equal to the actual number of degrees of freedom, by using a set **z** of independent coordinates. The starting point is to establish the following relation between velocities:

$$\dot{\mathbf{q}} = \mathbf{R}\dot{\mathbf{z}} \tag{7}$$

where **q** are all the n_d dependent variables and **z** is a set of n_i independent variables. Such a relation can always be found, for instance, by taking the derivative of the restrictions, $\mathbf{\Phi_q}\dot{\mathbf{q}} = \mathbf{0}$, and splitting all the velocities in two subsets, so that one subset of velocities may be written as a function of the other subset. Once (7) is obtained, it follows that

$$\ddot{\mathbf{q}} = \mathbf{R}\ddot{\mathbf{z}} + \dot{\mathbf{R}}\dot{\mathbf{z}} \tag{8}$$

Going back to (3), premultiplying by the transpose of **R**, and having in mind that $\mathbf{\Phi_q}\mathbf{R} = \mathbf{0}$,

$$\ddot{\mathbf{z}} = \left(\mathbf{R}^{\mathrm{T}}\mathbf{M}\mathbf{R}\right)^{-1}\left[\mathbf{R}^{\mathrm{T}}\left(\mathbf{Q} - \mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{z}}\right)\right] = \bar{\mathbf{M}}^{-1}\bar{\mathbf{Q}} \tag{9}$$

which defines the corrected mass matrix $\bar{\mathbf{M}}$ and the corrected vector of generalized forces $\bar{\mathbf{Q}}$. The result is that the DAE (3) in the dependent variables has been converted into the ODE (9) expressed in independent variables.

The main advantage of the matrix-**R** method is the reduction of the number of equations, at the expense of having to compute, at each instant, **R** and the dependent states as functions of the independent ones. It also requires the effort of managing the redundancy in restrictions and the changes in the representative set of velocities.

If now the states are defined as $\mathbf{x}^{\mathrm{T}} = \left\{\mathbf{z}^{\mathrm{T}} \;\; \mathbf{w}^{\mathrm{T}}\right\}$, with $\mathbf{w} = \dot{\mathbf{z}}$, the following equations can be written,

$$\begin{gathered} \dot{\mathbf{z}} = \mathbf{w} \\ \dot{\mathbf{w}} = \left(\mathbf{R}^{\mathrm{T}}\mathbf{M}\mathbf{R}\right)^{-1}\left[\mathbf{R}^{\mathrm{T}}\left(\mathbf{Q} - \mathbf{M}\dot{\mathbf{R}}\mathbf{w}\right)\right] = \bar{\mathbf{M}}^{-1}\bar{\mathbf{Q}} \end{gathered} \tag{10}$$

or, more compactly,

$$\begin{Bmatrix} \dot{\mathbf{z}} \\ \dot{\mathbf{w}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{w} \\ \bar{\mathbf{M}}^{-1}\bar{\mathbf{Q}} \end{Bmatrix} \;\Rightarrow\; \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \tag{11}$$

These equations perfectly match (1) and, therefore, the EKF in (2) can be straightforwardly applied. In particular, the state-space matrix is obtained as the linearization:

$$\mathfrak{A} = \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \dfrac{\partial\left(\bar{\mathbf{M}}^{-1}\bar{\mathbf{Q}}\right)}{\partial \mathbf{z}} & \dfrac{\partial\left(\bar{\mathbf{M}}^{-1}\bar{\mathbf{Q}}\right)}{\partial \mathbf{w}} \end{bmatrix} \tag{12}$$

which can be approximated as,

$$\begin{gathered} \mathfrak{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathfrak{A}_{21} & \mathfrak{A}_{22} \end{bmatrix} \\ \mathfrak{A}_{21} = -\bar{\mathbf{M}}^{-1}\mathbf{R}^{\mathrm{T}}\left(\mathbf{K}\mathbf{R} + 2\mathbf{M}\mathbf{R_q}\mathbf{R}\dot{\mathbf{w}}\right) \\ \mathfrak{A}_{22} = -\bar{\mathbf{M}}^{-1}\mathbf{R}^{\mathrm{T}}\left(\mathbf{C}\mathbf{R} + \mathbf{M}\dot{\mathbf{R}}\right) \end{gathered} \tag{13}$$

where **K** is the stiffness matrix and **C** the damping matrix.

In this case, the size of the problem is $2n_i$. Now, according to (2), the correction provided by the EKF must be included into the observer equations,

$$\begin{gathered} \dot{\mathbf{z}} - \mathbf{w} + \mathfrak{K}_1\left(\mathbf{y} - \mathbf{y}_s\right) = \mathbf{0} \\ \bar{\mathbf{M}}\dot{\mathbf{w}} - \bar{\mathbf{Q}} + \bar{\mathbf{M}}\mathfrak{K}_2\left(\mathbf{y} - \mathbf{y}_s\right) = \mathbf{0} \end{gathered} \tag{14}$$

where $\mathfrak{K}_1$ and $\mathfrak{K}_2$ are the upper and lower parts of the Kalman gain matrix $\mathfrak{K}$, and $\mathbf{y}_s$ are the outputs provided by the sensors.

Since real-time performance of the algorithms will be required by the final applications, the integration procedure is relevant in order to make the algorithm as efficient as possible. The implicit single-step trapezoidal rule has been selected as integrator,

$$\begin{gathered} \dot{\mathbf{z}}_{n+1} = \frac{2}{\Delta t}\mathbf{z}_{n+1} - \left(\frac{2}{\Delta t}\mathbf{z}_n + \dot{\mathbf{z}}_n\right) \\ \dot{\mathbf{w}}_{n+1} = \frac{2}{\Delta t}\mathbf{w}_{n+1} - \left(\frac{2}{\Delta t}\mathbf{w}_n + \dot{\mathbf{w}}_n\right) \end{gathered} \tag{15}$$

Now, (15) can be substituted into (14), thus leading to the nonlinear system of equations in the states,

$$\begin{cases} \mathbf{g}_1(\mathbf{x}_{n+1}) = \mathbf{0} \\ \mathbf{g}_2(\mathbf{x}_{n+1}) = \mathbf{0} \end{cases} \;\Rightarrow\; \mathbf{g}(\mathbf{x}_{n+1}) = \mathbf{0} \tag{16}$$

This system can be iteratively solved by the Newton-Raphson iteration, the approximated tangent matrix being,

$$\frac{\partial \mathbf{g}}{\partial \mathbf{x}} = \begin{bmatrix} \dfrac{2}{\Delta t}\mathbf{I} & -\mathbf{I} \\ \mathbf{R}^{\mathrm{T}}\mathbf{K}\mathbf{R} & \mathbf{R}^{\mathrm{T}}(\mathbf{C}\mathbf{R} + \mathbf{M}\dot{\mathbf{R}}) + \dfrac{2}{\Delta t}\bar{\mathbf{M}} \end{bmatrix} + \tag{17}$$

$$\begin{bmatrix} \mathbb{K}_1\mathbb{C}_1 & \mathbb{K}_1\mathbb{C}_2 \\ \bar{\mathbf{M}}\mathbb{K}_2\mathbb{C}_1 & \bar{\mathbf{M}}\mathbb{K}_2\mathbb{C}_2 \end{bmatrix}$$

where $\mathbb{C}_1$ and $\mathbb{C}_2$ are the upper and lower parts of the output Jacobian matrix $\mathbb{C}$.

5. Penalty formulation

The basic idea in the penalty method is to postulate that the constraining forces in (3) are proportional to the violation of the restrictions. In particular, the Lagrange multipliers are chosen in the form [6]:

$$\boldsymbol{\lambda} = \alpha\left(\ddot{\boldsymbol{\Phi}} + 2\zeta\omega\dot{\boldsymbol{\Phi}} + \omega^2\boldsymbol{\Phi}\right) \tag{18}$$

where α is the penalty factor, usually fixed to a very large value, 10^7 or more. Notice that the combination of the constraint functions and their derivatives takes the form of a second order oscillating system with damping coefficient and natural frequency usually chosen as ζ=1, ω=10. So, the rigid constraints in the DAE (3) can be converted into non-rigid constraints in an ODE:

$$\begin{aligned} \ddot{\mathbf{q}} = &\left(\mathbf{M} + \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\alpha\boldsymbol{\Phi}_{\mathbf{q}}\right)^{-1}\left[\mathbf{Q} - \right. \\ &\left. \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\alpha\left(\dot{\boldsymbol{\Phi}}_{\mathbf{q}}\dot{\mathbf{q}} + 2\zeta\omega\dot{\boldsymbol{\Phi}} + \omega^2\boldsymbol{\Phi}\right)\right] \end{aligned} \tag{19}$$

However, due to the very large value of α, it can be shown that this is equivalent to representing the constraints by springs of large stiffness, dampers of large friction coefficient and masses of large inertia. In this way, the constraints can actually be violated, but only in a very small amount, enough for representing de DAE (3) as the ODE (19) with negligible approximation errors.

Compared to the matrix-**R** method (9), the Eq. (19) has the drawback that the number of variables is larger: it is equal to the total number of dependent variables, n_d. However, this method has the advantage that (19) can be directly integrated as an ODE and it is not necessary to solve at each time instant the problems of passing from independent to dependent states and related problems mentioned in the previous section. Furthermore, the corrected mass matrix (the inverted matrix in (19)) is invertible, even if **M** is only positive semidefinite.

$$\begin{aligned} &\dot{\mathbf{q}} = \mathbf{v} \\ &\dot{\mathbf{v}} = (\mathbf{M} + \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\alpha\boldsymbol{\Phi}_{\mathbf{q}})^{-1}[\mathbf{Q} - \\ &\boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\alpha(\dot{\boldsymbol{\Phi}}_{\mathbf{q}}\mathbf{v} + 2\zeta\omega\dot{\boldsymbol{\Phi}} + \omega^2\boldsymbol{\Phi})] = \bar{\mathbf{M}}^{-1}\bar{\mathbf{Q}} \end{aligned} \tag{20}$$

or, more compactly,

$$\begin{Bmatrix} \dot{\mathbf{q}} \\ \dot{\mathbf{v}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{v} \\ \bar{\mathbf{M}}^{-1}\bar{\mathbf{Q}} \end{Bmatrix} \Rightarrow \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \tag{21}$$

Then, the EKF in (2) can be straightforwardly applied. In particular, the state-space matrix is obtained as the linearization:

$$\mathbb{A} = \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \dfrac{\partial\left(\bar{\mathbf{M}}^{-1}\bar{\mathbf{Q}}\right)}{\partial \mathbf{q}} & \dfrac{\partial\left(\bar{\mathbf{M}}^{-1}\bar{\mathbf{Q}}\right)}{\partial \mathbf{v}} \end{bmatrix} \tag{22}$$

which can be approximated as,

$$\begin{aligned} &\mathbb{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbb{A}_{21} & \mathbb{A}_{22} \end{bmatrix} \\ &\mathbb{A}_{21} = -\bar{\mathbf{M}}^{-1}\left(\mathbf{K} + \omega^2\boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\alpha\boldsymbol{\Phi}_{\mathbf{q}} + 2\boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\alpha\boldsymbol{\Phi}_{\mathbf{qq}}\dot{\mathbf{v}}\right) \\ &\mathbb{A}_{22} = -\bar{\mathbf{M}}^{-1}\left[\mathbf{C} + \boldsymbol{\Phi}_{\mathbf{q}}^{\mathrm{T}}\alpha\left(\dot{\boldsymbol{\Phi}}_{\mathbf{q}} + 2\zeta\omega\boldsymbol{\Phi}_{\mathbf{q}}\right)\right] \end{aligned} \tag{23}$$

In this case, the size of the problem is $2n_d$. As the size of the problem increases, it affects particularly the covariance matrix **P** (see (2)), with a number of entries proportional to the square of the size.

Now, according to (2), the correction provided by the EKF must be included into the observer equations,

$$\begin{aligned} &\dot{\mathbf{q}} - \mathbf{v} + \mathbb{K}_1\left(\mathbf{y} - \mathbf{y}_s\right) = \mathbf{0} \\ &\bar{\mathbf{M}}\dot{\mathbf{v}} - \bar{\mathbf{Q}} + \bar{\mathbf{M}}\mathbb{K}_2\left(\mathbf{y} - \mathbf{y}_s\right) = \mathbf{0} \end{aligned} \tag{24}$$

where $\mathbb{K}_1$ and $\mathbb{K}_2$ are the upper and lower parts of the Kalman gain matrix $\mathbb{K}$, and $\mathbf{y}_s$ are the outputs provided by the sensors.

As for the previous formulation, the implicit single-step trapezoidal rule has been selected as integrator,

$$\begin{aligned} \dot{\mathbf{q}}_{n+1} &= \frac{2}{\Delta t}\mathbf{q}_{n+1} - \left(\frac{2}{\Delta t}\mathbf{q}_n + \dot{\mathbf{q}}_n\right) \\ \dot{\mathbf{v}}_{n+1} &= \frac{2}{\Delta t}\mathbf{v}_{n+1} - \left(\frac{2}{\Delta t}\mathbf{v}_n + \dot{\mathbf{v}}_n\right) \end{aligned} \tag{25}$$

Now, (25) can be substituted into (24), thus leading to the nonlinear system of equations in the states,

$$\begin{cases} \mathbf{g}_1(\mathbf{x}_{n+1}) = \mathbf{0} \\ \mathbf{g}_2(\mathbf{x}_{n+1}) = \mathbf{0} \end{cases} \Rightarrow \quad \mathbf{g}(\mathbf{x}_{n+1}) = \mathbf{0} \tag{26}$$

This system can be iteratively solved by the Newton-Raphson iteration, the approximated tangent matrix being,

$$\frac{\partial \mathbf{g}}{\partial \mathbf{x}} = \begin{bmatrix} \frac{2}{\Delta t}\mathbf{I} & -\mathbf{I} \\ \mathbf{T}_{21} & \mathbf{T}_{22} \end{bmatrix} + \begin{bmatrix} \mathfrak{K}_1\mathfrak{C}_1 & \mathfrak{K}_1\mathfrak{C}_2 \\ \bar{\mathbf{M}}\mathfrak{K}_2\mathfrak{C}_1 & \bar{\mathbf{M}}\mathfrak{K}_2\mathfrak{C}_2 \end{bmatrix}$$

$$\mathbf{T}_{21} = \mathbf{K} + \omega^2 \mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}} \alpha \mathbf{\Phi}_{\mathbf{q}} \tag{27}$$

$$\mathbf{T}_{22} = \mathbf{C} + \mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}} \alpha \left(\dot{\mathbf{\Phi}}_{\mathbf{q}} + 2\zeta\omega \mathbf{\Phi}_{\mathbf{q}} \right) + \frac{2}{\Delta t}\bar{\mathbf{M}}$$

where $\mathfrak{C}_1$ and $\mathfrak{C}_2$ are the upper and lower parts of the output Jacobian matrix $\mathfrak{C}$.

6. Example

The four-bar mechanism with a spring-damper element shown in Fig. 1 is chosen as example. Two computational versions of the mechanism are created: the first one represents the real "prototype", while the second one plays the role of the "model". A sensor in the prototype provides as a measurement y=s, the distance between point A and point 2, i.e., the ends of the spring-damper element.

The state variables in this example are the Cartesian coordinates of points 1 and 2, and the distance s:

$$\mathbf{q}^{\mathrm{T}} = \{ x_1 \quad y_1 \quad x_2 \quad y_2 \quad s \} \tag{28}$$

The independent variable $\mathbf{z}$ for the matrix-$\mathbf{R}$ method is the distance s.

The mass matrix $\mathbf{M}$, the force vector $\mathbf{Q}$ and the constraints $\mathbf{\Phi}$ of the mechanism are obtained according to [6].

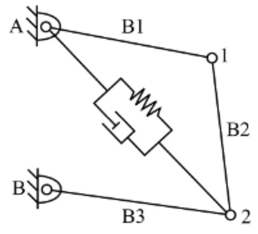

Fig. 1. Four-bar mechanism with spring-damper element.

The numerical values of the parameters are the masses m_{B1} =2 kg, m_{B2} =25 kg, m_{B3} =2 kg; bar lengths L_{B1} =0.9 m, L_{B2} =1 m, L_{B3} =1.05 m; natural spring length s_0 =1.4 m; fixed points Cartesian coordinates A=(0,0), B=(0,−1); spring coefficient k=10000 N/m, and damping coefficient c=500 Ns/m.

Regarding the tuning of the EKF, the main parameters are the matrices $\mathbf{\Theta}$, $\mathbf{\Xi}$. In principle, they have to represent the covariances of the zero-mean noises $\boldsymbol{\delta}$, $\boldsymbol{\varepsilon}$, in (1). However, in practice, this information is not always clearly known so that the matrices $\mathbf{\Theta}$, $\mathbf{\Xi}$, are used as tuning parameters adjusted by trial-and-error experimental work.

It should be stressed that there is no perfect solution to the filtering problem, but rather there exist trade-offs between competing objectives. Solutions that provide fast convergence are affected by higher levels of noise. If low levels of noise are desired, then the initial errors converge more slowly to zero. To facilitate the tuning, the matrices are postulated to be diagonal, $\mathbf{\Theta}$=diag(θ_i), $\mathbf{\Xi}$=diag(ξ_i).

After some trial-and-error test work under the simulation conditions to be detailed later, the EKF tuning is set to $\mathbf{\Theta}$=diag(10), $\mathbf{\Xi}$=diag(0.01). The initial covariance value has been chosen to be $\mathbf{P}$(0)=diag(diag(p_i),diag(v_i)), to represent different initial uncertainties in positions and velocities. As it is supposed that model and prototype start from rest conditions, the initial uncertainty is zero in velocities and, let us say, 0.1 in positions, so that $\mathbf{P}$(0)=diag (diag(0.1), diag(0)).

Regarding the simulation conditions, to show the recovery from different initial conditions, the real prototype starts at s(0)=1.80 m, while the observer starts at s(0)=1.85 m. To evaluate the effect of noise in measurements, it is supposed a sensor noise of 0.02

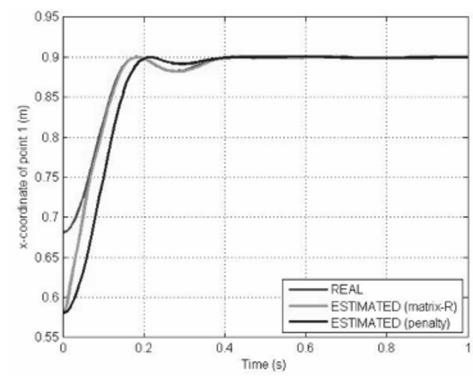

Fig. 2. History of state x_1 .

Table 1. CPU-TIMES (s).

Method	Matrix-R	Penalty	Penalty+
CPU-time (s)	0.11	0.09	0.18

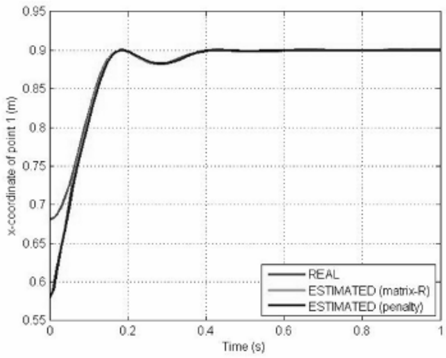

Fig. 3. History of state x_1 (position and velocity problems are now solved for the penalty method).

m is uniformly distributed. Finally, to check the effect of uncertain exogenous forces, the prototype runs under normal gravity, g=9.81 m/s^2, but the observer runs under g=8.81 m/s^2. In these conditions, the history of the state x_1 (real vs. estimated through both matrix-**R** and penalty method) is plotted in Fig. 2.

It can be seen in Fig. 2 that the observer based on the penalty method behaves worse than the one based on the matrix-**R** method. The reason is that, while the measured coordinate s is perfectly followed by the observer, the remaining coordinates x_1, y_1, x_2, y_2, are not consistent with the distance s, since the penalty terms are not capable of ensuring the constraint satisfaction under the large forces introduced by the EKF. And this happens for any value of the penalty factor.

The explanation to this phenomenon can be found by looking at the second equation in (24): an increment of the penalty factor increases the penalty forces which oppose to constraint violation but, at the same time, increases the value of the correction terms coming from the EKF. Therefore, increasing the value of the penalty factor so as to guarantee constraint satisfaction is worthless.

Hence, to ensure constraint satisfaction, the kinematic position and velocity problems should be solved at each function evaluation for the penalty method. Fig. 3 shows that the obtained results in that case are as good as those provided by the matrix-**R** method, at the cost of losing the advantages of the penalty method pointed out in Section 5.

Regarding the efficiency, the CPU-times required for both methods to run the described 1 s simulation under Matlab environment are gathered in Table 1. The fixed time-step used for the numerical integration is, in all cases, Δt=0.01 s. The method "Penalty+" in Table 1 refers to the penalty method when the position and velocity problems are solved (Fig. 3).

Therefore, it is clear that the matrix-**R** method is more efficient than the penalty method, due to the above mentioned need of solving the position and velocity problems in the penalty method in order to ensure constraint satisfaction.

7. Conclusions

This work presents a study on the application of EKF observers to multibody systems. Although the numerical tests have been carried out on a simple four-bar example, the final objective is to implement the observers on complex multibody systems for advanced real-time applications.

The approach has been based on the idea that the same techniques that are suitable for fast simulation of multibody systems will be efficient as well for implementing the observers. Two methods have been chosen: the matrix-**R** formulation and the penalty formulation. The detailed development of the EKF observer for these two methods has been presented.

The simulation tests show successful results. After a not very involved trial-and-error tuning, the EKF observers are robust with respect to sensor noise and errors in physical parameters, initial position and actuation readings.

The matrix-**R** method has the advantage of leading to a lower problem size. The penalty method, whose corrected matrix is always invertible, was expected to have the advantage of neither requiring the computation of the dependent states as functions of the independent ones, nor the effort of managing the redundancy in constraints and the changes in the representative set of velocities. However, it was found that constraint satisfaction was not achieved by the penalty terms and, therefore, the mentioned advantage was lost. Consequently, the matrix-**R** method showed to be more efficient.

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